* S B - 3 5 3 8 / 2 0 0 *

SB-3538

M. Sc. - II Examination

March / April – 2011

Paper - 5006 : Mathematics

(Advance Integral Transform)

Time : Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fillup strictly the details of ☛ signs on your answer book.**	Seat No. :
Name of the Examination : ☛ **M. Sc. - II**	
Name of the Subject : ☛ **Peper - 5006 : Mathematics**	
☛ Subject Code No. : **3 5 3 8** ☛ Section No. (1, 2,.....) : **Nil**	Student's Signature

(2) Attempt all questions.

(3) Notations used are standard.

1 (a) Obtain the solution of the boundary value problem **6**

$y^2 \frac{\partial u}{\partial x^2} + y\frac{\partial u}{\partial y} + \frac{\partial^2 u}{\partial x^2} = 0$ for $0 < y < \infty$ with boundary conditions u(y,0)=0

$$u(y,1) = \begin{cases} C; o \le y \le 1 \\ 0; y > 1 \end{cases}.$$

(b) Define inverse Mellin transform. Also state and prove the convolution theorem for inverse Mellin transform. **4**

(c) Prove that $M\left[f^n(x):s\right] = \frac{(-1)^n (s-1)}{[(s-n)-1]!} f^*(s-n)$. **4**

OR

1 (a) Prove that $M\left[\int_0^\infty f(xu)g(u)du : s\right] = f^*(s)g^*(1-s)$. Also **6**

solve the integral equation $\int_0^\infty f(\xi) g\left(\frac{x}{\xi}\right)\frac{d\xi}{\xi} = h(x)$; x>0.

(b) In usual notation prove that $M\left[\frac{\overline{)a}}{(1+x)^a}:S\right]=\overline{)a-s}\overline{)s}$. **4**

(c) If $M[f(x)]=\overline{f}(s)$, then **4**

$\sum_{n=0}^{\infty} f(n+a)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\overline{f}(s)\xi(s,a)ds$, where $\xi(s,a)$ is the Hurwitz zeta function defined by $\xi(s,a)=\sum_{n=0}^{\infty}\frac{1}{(n+a)^s}$;

$0\le a\le 1$, $\mathrm{Re}(s)>1$.

2 (a) Heat is supplied at a constant rate Q per area and per unit time over a circular area of radius 'a' in the plane z=0 to an infinite solid of conductivity 'K_1'. Show that a steady temperature at a distance 'r' from a α is of circular area and distance z from the plane z=0 is given by, **6**

$$v=\frac{Qa}{2K_1}\int_0^{\infty} e^{-kz}J_1(ak)J_0(kr)k^{-1}dk.$$

(b) Find Hankel transform of $f(r)=\begin{cases} a^2-r^2; o<r\le r \\ 0; r\ge a \end{cases}; n=0$. **4**

(c) Define Hankel transform and find : **4**

(i) $H_0\left[\frac{e^{-ar}}{r}:k\right]$

(ii) $H_1\left[r^{-2}e^{r}:k\right]$

(iii) $H_1\left[r^{-2}e^{-r}:k\right]$.

OR

2 (a) Apply Hankel transform of order zero to solve differential equation $\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}+\frac{\partial^2 v}{\partial z^2}=0$; $r>0$ satistfying following conditions $v\to 0$ as $z\to\infty$, $r\to\infty$ and v=f(r); z=0; $r\ge 0$. **6**

(b) Prove that $H_n\left[e^{-\frac{1}{4}px^2} f(x):k\right] = 2^{n+1}$ **4**

$L\left[x^{\frac{n}{2}} J_n(2k\sqrt{x}):p\right]$ when $f(x) = x^n$.

(c) Evaluate : **4**

(i) $H_1\left[\frac{e^{-ar}}{r}:k\right]$

(ii) $H_1^{-1}\left[k^2 e^{-ak}:r\right]$

(iii) $H_0^{-1}\left[e^{-ak}:r\right]$

(iv) $H_0^{-1}\left[\frac{e^{-ak}}{k}:r\right]$.

3 (a) Use finite Hankel transform to solve differential equation $\frac{\partial^2 v}{\partial r^2} + \frac{1\partial v}{r\partial r} = \frac{1}{k_1}\frac{\partial v}{\partial t}$; $0 \le r \le 1$; $t > 0$ where $\frac{\partial v}{\partial r} + hv = 0$; r=1, t>0 and v=1; t=0, $0 \le r \le 1$. **6**

(b) Find the finite Hankel transform of order zero of 'r^2'. **4**

(c) Find finite Hankel transform of order zero of a constant function. **4**

OR

3 (a) Use the finite Hankel transform to solve the following differential equation $\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} = \frac{1}{k_1}\frac{\partial v}{\partial t}$; $0 \le r \le 1$; t>0 **6**

where

$v = v_o =$ constant; r=1; t>0

v=0; t=0; $0 \le r \le 1$.

(b) Prove that finite Hankel transform of order 'n' of **4**

$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}$ is $\frac{k}{2}\left[H_{n+1}^*\left(\frac{\partial f}{\partial r}\right) - H_{n-1}^*\left(\frac{\partial f}{\partial r}\right)\right]$.

(c) Find the finite Hankel transform of r^{n-1} where $rJ_{n-1}(kr)$ is the kernel of transform. 4

4 (a) Find the Z-transform of $\cosh(n\theta)$ and $\sinh(n\theta)$. 6

(b) State and prove the shifting theorem for Z-transform. 4

(c) State and prove the damping the Z-transform. 4

OR

4 (a) If $\overline{U}(z)=\dfrac{2z^2+3z+12}{(z-1)^4}$ find the values of U_2 and U_3. 6

(b) Using Z-transform solve $U_{n+2}-2U_{n+1}+U_n=2^n$ given that $U_0=2$ and $U_1=1$. 4

(c) Find : 4

(i) $Z^{-1}\left[\dfrac{3z^2-z}{(z-1)(z-2)^2}\right]$ and

(ii) $Z^{-1}\left[\dfrac{z}{(z-1)(z-2)}\right]$.

5 (a) Define stieltjes transform also derive the formula for stieltjes transform. 6

(b) For stieltjes transform (S_t), show that : 4

$S_t(\sin(k\sqrt{t}))=\pi\exp(-k\sqrt{z})$.

(c) For generalized stieltjes transform prove that : 4

$S_g(e^{-(at)})=e^{az}a^{\rho-1}\sqrt{\rho-1}$.

OR

5 (a) Derive the formula for inverse Hilbert transform. 6

(b) Find the Hilbert transform $\sin(\omega t)$. 4

(c) For stieltjes transform (S_t), prove that 4

$S_t(f\sqrt{t})=\overline{f}(i\sqrt{z})+\overline{f}(-i\sqrt{z})$.
